Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Course: Mathematics 1 – B Date: May 18 , 2019 Duration: **3** hours

The Exam consists of two papers   Answer **All** Questions   No. of questions: 4   Total Mark: 100

Question 1

(a)State the definition of : (i)Parabola   (ii)Ellipse   (iii)Plane — 6

(b)Describe the surfaces: $x^2+y^2+z^2-x=0,\ x^2+z^2+4y=0,\ x^2-y^2-z^2=0.$ — 6

(c)Show that the equation $2xy-2x-1=0$ is hyperbola. Eliminate the xy-term and sketch its curve. Also, find its conjugate hyperbola. — 6

(d)Determine the center, vertices and sketch the ellipse $4x^2+y^2-8x+2y+1=0$. — 4

(e)Find the distance between the two lines $x=\frac{y-1}{2}=\frac{z+1}{2}$ and $x-1=\frac{y-2}{2}=\frac{z+2}{2}$ and write the equation of the plane containing them. — 4

(f)Show that the two lines $\frac{x-4}{2}=\frac{y-2}{-2}=\frac{z-1}{1}$ and $\frac{x}{2}=\frac{y-3}{2}=\frac{z-1}{-1}$ are skew and find the short distance between them. — 4

Question 2

(a)Prove that the line $2x+y=4$ is a tangent to the circle $x^2+y^2+6x-10y+29=0$. — 4

(b)Determine the vertex, the focus, equation of the directrix, length of latus rectum and sketch the curve of the parabola $x^2-2x+4y+9=0$. — 5

(c)Find the value of k such that the equation $x^2+2xy-3y^2+4x+ky+4=0$ represents pair of lines and then separate them. Find their point of intersection, the angle between them and the bisectors. — 5

(d)If the point $(3,2)$ is limiting point of a system of coaxial circles where the circle $x^2+y^2-14x+2y=0$ is one of this system. Find the equation of that system and the second limit point. — 6

Question 3

(a)Find the circumference of asteroid and the area bounded by the lemniscates: $r=a\sqrt{\cos 2\theta}$ — 10

(b)Find the integrals (i) $\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos x\,.\ln\frac{1+x}{1-x}dx$ (ii) $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ (iii) $\int_0^3\frac{dx}{(x-1)^{\frac{2}{3}}}dx$ — 9

(c)Determine the volume of revolution obtained by rotating the region bounded by $y=\sqrt[3]{x}$ *and* $y=\frac{x}{4}$ that lies in the first quadrant about the y-axis. — 6

إنظر السؤال الرابع بالورقة الثانية

Good Luck : Dr. Mohamed Eid, Dr. Khaled Elnaggar , Dr. Reda Abd Elkader

Model Answer

Answer of Question 1

(a)Definitions: (i) Parabola (ii)Ellipse (iii)Plane

-- ***(6 Marks)***

(b)Sphere: $x^2 + y^2 + z^2 - x = 0$ is written as: $(x - \frac{1}{2})^2 + y^2 + z^2 = \frac{1}{4}$, its center (1/2, 0,0) and its radius is 1/2.

Paraboloid: $x^2 + z^2 + 4y = 0$ with vertex (0, 0, 0), its axis is the negative y-axis.

Cone: $x^2 - y^2 - z^2 = 0$ with vertex (0, 0, 0), its axis is the x-axis.

-- ***(6 Marks)***

(c) From the equation $2xy - 2x - 1 = 0$, $\Delta = \begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & -1 \end{vmatrix} = 1$ and $h^2 = 1 > a.b = 0$.

Then its hyperbola. Since $\tan 2\theta = \frac{2h}{a-b} = \frac{2}{0}$. Then $2\theta = \frac{\pi}{2}$ and $\theta = \frac{\pi}{4}$

Then from the substitution $x = X\cos\frac{\pi}{4} - Y\sin\frac{\pi}{4}$ and $y = X\sin\frac{\pi}{4} + Y\cos\frac{\pi}{4}$

Then $x = \frac{X-Y}{\sqrt{2}}$ and $y = \frac{X+Y}{\sqrt{2}}$

Then $\frac{X-Y}{\sqrt{2}} . \frac{X+Y}{\sqrt{2}} - 2\frac{X-Y}{\sqrt{2}} - 1 = 0$ Or $(X - \frac{1}{\sqrt{2}})^2 - (Y - \frac{1}{\sqrt{2}})^2 = 1$.

It is vertical with center $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and a = b = 1.

The asymptotic lines is given by: $2xy - 2x + C = 0$ and $\Delta = \begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & c \end{vmatrix} = -C = 0$

Then, the asymptotic lines is given by: $2xy - 2x = 0$ and the conjugate hyperbola is

$2xy — 2x - 1 = 0$

-- ***(6 Marks)***

(d)The ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$ is written as $4(x-1)^2 + (y+1)^2 = 4$

It is vertical with center (1, –1), a = 1, b = 2.

-- ***(4 Marks)*** (e)We see that $\vec{u} = \vec{v} = i + 2j + 2k$.

Then, the two lines $x = \frac{y-1}{2} = \frac{z+1}{2}$ and $x - 1 = \frac{y-2}{2} = \frac{z+2}{2}$ are parallels.

Since $P_1 = (0, 1, -1)$, $P_2 = (1, 2, -2)$. Then $\overrightarrow{P_1P_2} = i + j - k$

Then $\overrightarrow{P_1P_2} \times \vec{u} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & 2 & 2 \end{vmatrix} = 4i - 3j + k = \vec{N}$

Then the plane is : $4x - 3(y - 1) + z + 1 = 0$ Or $4x - 3y + z + 4 = 0$

The distance between the lines is: $D = \frac{|\overrightarrow{P_1P_2} \times \vec{u}|}{|\vec{u}|} = \frac{\sqrt{26}}{3}$

-- ***(4 Marks)***

(f)From the two lines $\frac{x-4}{2} = \frac{y-2}{-2} = \frac{z-1}{1}$ and $\frac{x}{2} = \frac{y-3}{2} = \frac{z-1}{-1}$

We see that $\vec{u} = 2i - 2j + k$, $\vec{v} = 2i + 2j - k$ and $P_1 = (4, 2, 1)$, $P_2 = (0, 3, 1)$.

Then $\overrightarrow{P_1P_2} = -4i + j + 0k$

Then $\overrightarrow{P_1P_2} \times \vec{u} = \begin{vmatrix} i & j & k \\ -4 & 1 & 0 \\ 2 & -2 & 1 \end{vmatrix} = i + 4j + 6k$

Then $\overrightarrow{P_1P_2} \times \vec{v} = \begin{vmatrix} i & j & k \\ -4 & 1 & 0 \\ 2 & 2 & -1 \end{vmatrix} = -i - 4j + 10k$

Then the two lines are skew.

Since $\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 2 & 2 & -1 \end{vmatrix} = 0i + 4j + 8k$ and $\overrightarrow{P_1P_2}.(\vec{u} \times \vec{v}) = 0 + 4 + 0 = 4$

Then the short distance is : $L = \frac{\overrightarrow{P_1P_2}.(\vec{u} \times \vec{v})}{|\vec{u} \times \vec{v}|} = \frac{4}{\sqrt{80}} = \frac{1}{\sqrt{5}}$

To obtain the end points M, N of the short distance, we write the lines in parametric form:

The first : $x = 2t + 4$, $y = -2t + 2$, $z = t + 1$, t in R

The second : $x = 2r$, $y = 2r + 3$, $z = -r + 1$, r in R

Then $\overrightarrow{MN} = (2r - 2t - 4)i + (2r + 2t + 1)j + (-r - t)k$

Since $\overrightarrow{MN}.\vec{u} = 2(2r - 2t - 4) - 2(2r + 2t + 1) + (-r - t) = 0$, $-r - 9t - 10 = 0$

$\overrightarrow{MN}.\vec{v} = 2(2r - 2t - 4) + 2(2r + 2t + 1) - (-r - t) = 0$, $9r + t - 6 = 0$

Then by solving the equations: $9t + r = -10$, $t + 9r = 6$. We get $t = -\frac{6}{5}$, $r = \frac{4}{5}$.

Then the two points are $M\left(\frac{8}{5}, \frac{22}{5}, -\frac{1}{5}\right)$, $N(\frac{8}{5}, \frac{23}{5}, \frac{1}{5})$.

The distance between them is $\sqrt{\frac{5}{25}} = \frac{1}{\sqrt{5}}$

-- ***(4 Marks)***

Dr. Mohamed Eid

Final Exam and ILOs

Course Title: Mathematics 1-B **Code:** EMP 021

Questions	**ILOs**		
	Knowledge and Understanding	Intellectual Skills	Professional and Practical Skills
	a.1	b.1	c.1
Q1	√	√	
Q2	√	√	√
Q3	√		√
Q4	√	√	

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